

SECTION 11

CALCULATION OF PRECISION LIGHT TIMES AND QUASAR DELAYS

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11.1 INTRODUCTION

This section gives the formulation for calculating precision values of the one-way light time ρ_1 , the round-trip light time ρ , and the quasar delay τ . There are two versions of the precision one-way light time. One is used to calculate computed values of one-way doppler (F_1) observables and one-way wideband (*IWS*) and narrowband (*INS*) spacecraft interferometry observables. The other precision one-way light time is used to calculate computed values of GPS/TOPEX pseudo-range and carrier-phase observables. The round-trip light time ρ is two-way if the transmitter (a tracking station on Earth or an Earth satellite) is also the receiver. If the receiver is not the transmitter, the round-trip light time is three-way. The precision light times ρ_1 , ρ , and τ are used in the formulation of Section 13 to calculate computed values of the observables.

Prior to discussing the calculation of the precision light times, a model which has been ignored so far in the formulation must be introduced. This is the model for the down-leg delay τ_D at the receiving station on Earth and the up-leg delay τ_U at the transmitting station on Earth. This model is necessary to process tracking data obtained at the new 34-m beam wave guide (BWG) antennas at the Goldstone complex. For these tracking stations, the transmitting and receiving electronics are located at a central site which is tens of kilometers away from the individual antennas. The model for representing τ_D and τ_U is given in Section 11.2.

The formulation for calculating the precision round-trip light time ρ is given in Section 11.3. The definition of ρ is given in Section 11.3.1. The formulation for computing ρ is given in Section 11.3.2.

Section 11.4 gives the formulation for calculating the precision one-way light time ρ_1 used to calculate the computed values of one-way doppler (F_1) observables and one-way wideband (*IWS*) and narrowband (*INS*) spacecraft interferometry observables. The computed values of each of these observables (each of the two computed F_1 observables which are differenced to obtain the computed *INS* observable) should be calculated from the differenced one-way

light time $\hat{\rho}_1$ defined to be the reception time $t_3(\text{ST})$ in station time ST at the receiver (a tracking station on Earth or an Earth satellite) minus the transmission time $t_2(\text{TAI})$ in atomic time TAI at the spacecraft. However, in order to calculate $\hat{\rho}_1$, an expression for calculating ET – TAI at the spacecraft is required. Since such a general expression is not available, we calculate the precision one-way light time ρ_1 , which is defined to be $t_3(\text{ST})$ minus $t_2(\text{ET})$ instead of $\hat{\rho}_1$. Then, the differenced one-way light time $\hat{\rho}_1$ is calculated as the differenced one-way light time ρ_1 plus the correction term Δ . The term Δ is defined to be the change in ET – TAI which occurs during the spacecraft transmission interval. The preceding discussion for calculating the differenced one-way light time is given in detail in Section 11.4.1. The tedious formulation for calculating Δ is given in Sections 11.4.2 and 11.4.3. The formulation for calculating the precision one-way light time ρ_1 is given in Section 11.4.4.

The formulation for calculating the precision one-way light time ρ_1 used to calculate the computed values of GPS/TOPEX pseudo-range and carrier-phase observables is given in Section 11.5. This version of ρ_1 is defined in Section 11.5.1. The formulation for computing ρ_1 is given in Section 11.5.2. The expression for ρ_1 includes the geometrical phase correction $\Delta\Phi$, which is the lag in the measured phase at the receiver due to the rotation of the receiver relative to the transmitter. The formulation for calculating $\Delta\Phi$ is given in Section 11.5.3. The formulation for calculating the variable part of the phase-center offset at the transmitting GPS satellite, the receiving TOPEX satellite, and the GPS receiving station on Earth is given in Section 11.5.4.

The formulation for calculating the precision quasar delay τ is given in Section 11.6. The definition of τ is given in Section 11.6.1. The formulation for computing τ is given in Section 11.6.2.

11.2 DELAYS

For round-trip spacecraft data types, the downlink delay τ_D at the receiving station on Earth and the uplink delay τ_U at the transmitting station on Earth are placed on the record of the OD file for the data point. For one-way spacecraft data types, only τ_D is given. For narrowband and wideband spacecraft

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and quasar interferometry data types, τ_D is given for each of the two receiving stations on Earth. For quasar interferometry data types, there is no τ_U . For round-trip spacecraft interferometry data types, the effect of τ_U at the transmitting station on Earth cancels to sufficient accuracy in calculating these differenced data types. Hence, for this case, τ_U is set to zero.

If the received signal at a tracking station on Earth is a carrier-arrayed signal obtained by combining signals from several antennas, it will contain a fixed delay on the order of 1 ms. This delay will be added to the down-leg station delay τ_D .

If a tracking station on Earth has its own transmitting and receiving electronics located close to the antenna, the values of τ_D and τ_U for that tracking station are very small. Small values of τ_D and τ_U are subtracted from the observed values of range observables and the values placed on each record of the OD file for that tracking station are set to zero.

If a transmitting station or a receiving station is an Earth satellite, the values of τ_D and τ_U for that station are currently set to zero.

In the following, a receiver or a transmitter can be a tracking station on Earth or an Earth satellite. Reference will be made to the reception time or the transmission time at the tracking point of the antenna at the receiver or the transmitter. At a DSN tracking station on Earth, the tracking point is the secondary axis of the antenna (see Section 10.5.1). If the receiver or the transmitter is an Earth satellite, the tracking point is the center of mass of the satellite or the nominal phase center of the receiving or transmitting antenna of the satellite. When the satellite ephemeris is interpolated for the position vector of the satellite, the position vector obtained can refer to either of these points (See Section 7.3.3, Step 3 and Section 8.3.6, Steps 2, 9, and 22). If a receiving station on Earth is a GPS receiving station, the tracking point of the antenna is the nominal phase center of the receiving antenna. The position vector of the nominal phase center is calculated as described in Section 7.3.1, Step 3a.

Section 11.2.1 gives the equations for calculating the reception time $t_3(\text{ST})_R$ at the receiving electronics (subscript R) at the receiver for each light-time solution for each spacecraft data type. It also gives the equations for calculating the reception time $t_1(\text{ST})_R$ at the receiving electronics at receiver 1 for each light-time solution for quasar interferometry data types. These are the equations of Section 10.2.3.3.1 with a subscript R added to the reception times. These equations are functions of the data time tag (TT) and the count time (if any) for the data point.

Section 11.2.2 gives the equations for transforming reception times at the receiving electronics (subscript R) to reception times (τ_D seconds earlier) at the tracking point of the receiver. These equations apply at the reception time t_3 for spacecraft light-time solutions and at the reception time t_1 at receiver 1 for quasar light-time solutions.

Section 11.2.3 gives the equation for transforming the transmission time at the tracking point of the transmitter for a spacecraft light-time solution to the transmission time at the transmitting electronics (subscript T) (τ_U seconds earlier). It also gives the equation for transforming the reception time at the tracking point at receiver 2 for a quasar light-time solution to the reception time at the receiving electronics (subscript R) (τ_D seconds later).

The equations in Sections 11.2.1 and 11.2.2 are used to calculate the reception time $t_3(\text{ST})$ at the tracking point of the receiver for each spacecraft light-time solution. This epoch is used in Section 8.3.6, Step 1, to start each spacecraft light solution. The equations in Sections 11.2.1 and 11.2.2 are also used to calculate the reception time $t_1(\text{ST})$ at the tracking point of receiver 1 for each quasar light-time solution. This epoch is used in Section 8.4.3, Step 1, to start each quasar light solution.

From the preceding paragraphs, it is seen that the down-leg delay τ_D at each receiver for spacecraft data types and the down-leg delay τ_D at receiver 1 for quasar data types affect the spacecraft and quasar light-time solutions. It will be seen in Sections 11.3 to 11.6 that the down-leg delay τ_D at each receiver and

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the up-leg delay τ_U at each transmitter affect the calculated precision values of the one-way light time ρ_1 , the round-trip light time ρ , and the quasar delay τ .

It will be seen in Section 13 that computed values of many of the data types are explicit functions of reception times at the receiver and transmission times at the transmitter. In each case, the reception times are at the receiving electronics (subscript R), and the transmission times are at the transmitting electronics (subscript T).

The spacecraft transponder delay is normally subtracted from range observables in the ODE. It is not modelled in program Regres. Sometimes, it is not subtracted in the ODE and is added to the computed values of range observables using CSP commands in the Regres editor (see Section 10.2). If a spacecraft has multiple transponders, each transponder will, in general, have a different delay.

11.2.1 TRANSFORMING DATA TIME TAG TO RECEPTION TIME(S) AT RECEIVING ELECTRONICS

Equations (10–33) to (10–35) of Section 10.2.3.3.1 give the reception time $t_3(\text{ST})$ of the spacecraft signal at the receiver for each calculated light-time solution for each spacecraft data type. Equations (10–36) and (10–37) give the reception time $t_1(\text{ST})$ of the quasar wavefront at receiver 1 for the quasar light-time solution for an *IWQ* observable and for each of the two light-time solutions for an *INQ* observable. Each reception time calculated from Eqs. (10–33) to (10–37) should have a subscript R, indicating that the reception time is specifically at the receiving electronics.

11.2.2 CALCULATING DELAYS AT THE BEGINNING OF SPACECRAFT AND QUASAR LIGHT-TIME SOLUTIONS

For a spacecraft light-time solution, given the reception time $t_3(\text{ST})_R$ in station time ST at the receiving electronics, calculated from one of Eqs. (10–33) to (10–35), and the down-leg delay τ_D at the receiver, the reception time $t_3(\text{ST})$ at the tracking point of the receiver is given by:

$$t_3(\text{ST}) = t_3(\text{ST})_{\text{R}} - \tau_{\text{D}} \quad \text{s} \quad (11-1)$$

The spacecraft light-time solution (Section 8.3.6) starts with this epoch.

For a quasar light-time solution, given the reception time $t_1(\text{ST})_{\text{R}}$ in station time ST at the receiving electronics at receiver 1, calculated from Eq. (10-36) or (10-37), and the down-leg delay τ_{D_1} at receiver 1, the reception time $t_1(\text{ST})$ at the tracking point of receiver 1 is given by:

$$t_1(\text{ST}) = t_1(\text{ST})_{\text{R}} - \tau_{\text{D}_1} \quad \text{s} \quad (11-2)$$

The quasar light-time solution (Section 8.4.3) starts with this epoch.

11.2.3 CALCULATING DELAYS AT THE END OF SPACECRAFT AND QUASAR LIGHT-TIME SOLUTIONS

For a spacecraft light-time solution, given the transmission time $t_1(\text{ST})$ in station time ST at the tracking point of the transmitter, calculated in Step 32 of the spacecraft light-time solution (Section 8.3.6), and the uplink delay τ_{U} at the transmitter, the transmission time $t_1(\text{ST})_{\text{T}}$ at the transmitting electronics is given by:

$$t_1(\text{ST})_{\text{T}} = t_1(\text{ST}) - \tau_{\text{U}} \quad \text{s} \quad (11-3)$$

For a quasar light-time solution, given the reception time $t_2(\text{ST})$ in station time ST at the tracking point of receiver 2, calculated in Step 15 of the quasar light-time solution (Section 8.4.3), and the downlink delay τ_{D_2} at receiver 2, the reception time $t_2(\text{ST})_{\text{R}}$ at the receiving electronics at receiver 2 is given by:

$$t_2(\text{ST})_{\text{R}} = t_2(\text{ST}) + \tau_{\text{D}_2} \quad \text{s} \quad (11-4)$$

11.3 PRECISION ROUND-TRIP LIGHT TIME ρ

Section 11.3.1 gives the definition of the precision round-trip light time ρ and Section 11.3.2 gives the equation for calculating ρ as a sum of terms. Most of the terms in this equation are calculated in the spacecraft light-time solution and in related calculations. Calculating ρ as a sum of terms instead of the difference of two epochs reduces the roundoff errors in this calculation by approximately four orders of magnitude.

11.3.1 DEFINITION OF ρ

The definition of the precision round-trip light time ρ is given by:

$$\rho = t_3(\text{ST})_{\text{R}} - t_1(\text{ST})_{\text{T}} \quad \text{s} \quad (11-5)$$

where $t_3(\text{ST})_{\text{R}}$ is the reception time in station time ST of the spacecraft signal at the receiving electronics at the receiver and $t_1(\text{ST})_{\text{T}}$ is the corresponding transmission time in station time ST at the transmitting electronics at the transmitter. The receiver and the transmitter can each be a tracking station on Earth or an Earth satellite. If the transmitter is the receiver, the round-trip light time ρ is called two-way. Otherwise, it is called three-way.

Substituting Eqs. (11-1) and (11-3) into Eq. (11-5) gives:

$$\rho = [t_3(\text{ST}) - t_1(\text{ST})] + \tau_{\text{D}} + \tau_{\text{U}} \quad \text{s} \quad (11-6)$$

where $t_3(\text{ST})$ is the reception time in station time ST at the tracking point of the antenna at the receiver and $t_1(\text{ST})$ is the transmission time in station time ST at the tracking point of the antenna at the transmitter. The various tracking points are defined in the fifth paragraph of Section 11.2. The quantity τ_{D} is the downlink delay at the receiver and τ_{U} is the uplink delay at the transmitter. The previously given definition of ρ is Eq. (10-17), which is the first term of Eq. (11-6). This previous definition was given prior to the introduction of delays in Section 11.2. The first term of Eq. (11-6) is the round-trip light time in station time ST calculated in the spacecraft light-time solution.

11.3.2 CALCULATION OF ρ

The precision round-trip light time ρ defined by Eq. (11-5) or (11-6) is calculated as the following sum of terms:

$$\begin{aligned} \rho = & \frac{r_{23}}{c} + RLT_{23} + \frac{r_{12}}{c} + RLT_{12} \\ & - (ET - TAI)_{t_3} + (ET - TAI)_{t_1} \\ & - (TAI - UTC)_{t_3} + (TAI - UTC)_{t_1} \\ & - (UTC - ST)_{t_3} + (UTC - ST)_{t_1} \quad \text{s} \quad (11-7) \\ & + \frac{1}{10^3 c} [R_c + \Delta_A \rho(t_3) + \Delta_{SC} \rho_{23} + \Delta_A \rho(t_1) + \Delta_{SC} \rho_{12}] \\ & + \tau_D + \tau_U \end{aligned}$$

where c is the speed of light in kilometers per second.

The down-leg range r_{23} , up-leg range r_{12} , down-leg relativistic light-time delay RLT_{23} , up-leg relativistic light-time delay RLT_{12} , the three time differences at the reception time t_3 , and the three time differences at the transmission time t_1 are all calculated in the round-trip spacecraft light-time solution as specified in Section 8.3.6.

In Eq. (11-7), the intermediate time UTC (Coordinated Universal Time) is only used when the receiver or the transmitter is a DSN tracking station on Earth. If the receiver is an Earth satellite, the intermediate time UTC is replaced with TOPEX master time (denoted as TPX). If the transmitter is an Earth satellite, the intermediate time UTC is replaced with GPS master time (denoted as GPS). Note that the constant values of $TAI - TPX$ and $TAI - GPS$ are obtained from the GIN file. The use of different inputs for the receiving and transmitting satellites allows for different constant offsets from satellite TAI (see Section 2.2.2) to the nominal values of station time ST at the two satellites.

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The parameter R_c is a solve-for round-trip range bias in meters. It is specified by the receiving DSN tracking station number and time block for that station.

The terms $\Delta_A \rho(t_3)$ and $\Delta_A \rho(t_1)$ are antenna corrections at receiving and transmitting DSN tracking stations on Earth, calculated from the formulation of Section 10.5. They are a function of the antenna type at the DSN tracking station, the axis offset b , and the secondary angle of the antenna. The value of this angle used to evaluate each antenna correction is one of the unrefracted auxiliary angles calculated at t_3 or t_1 from the formulation of Section 9. If the receiver or the transmitter is an Earth satellite, the analogous correction is the offset from the center of mass of the satellite to the nominal phase center of the satellite. This offset is calculated as described in Section 7.3.3 when interpolating the ephemeris of the satellite.

The down-leg solar corona range correction $\Delta_{SC} \rho_{23}$ and the up-leg solar corona range correction $\Delta_{SC} \rho_{12}$ are calculated in the spacecraft light-time solution from the formulation of Section 10.4.

The down-leg delay τ_D at the receiver and the up-leg delay τ_U at the transmitter are obtained from the record of the OD file for the data point.

Equation (11-7) does not include corrections due to the troposphere or due to charged particles. These corrections are calculated in the Regres editor and are included in Eqs. (10-27) to (10-29) for the corrections $\Delta \rho$, $\Delta \rho_e$, and $\Delta \rho_s$ to ρ given by Eq. (11-7). These corrections to ρ are handled separately as described in Sections 10.1 and 10.2.

In order to minimize roundoff errors in the precision round-trip light time ρ calculated from the sum of terms (11-7), add the terms r_{23}/c and r_{12}/c to the sum last.

11.4 PRECISION ONE-WAY LIGHT TIME ρ_1

This section gives the formulation for calculating the differenced one-way light time $\hat{\rho}_1$ which is used to calculate the computed values of one-way doppler

(F_1) observables and one-way narrowband (*INS*) and wideband (*IWS*) spacecraft interferometry observables. The high-level equations for calculating the differenced one-way light time are given in Section 11.4.1. Calculation of the differenced one-way light time requires the calculation of the quantity Δ , which is the change in the time difference $ET - TAI$ which occurs during the transmission interval at the spacecraft. The high-level equations for calculating Δ are given in Section 11.4.2. The detailed algorithm for calculating the arguments of the quantity Δ is given in Section 11.4.3. The expression for the precision one-way light time ρ_1 , which does not include the time difference $ET - TAI$ at the transmission time t_2 at the spacecraft, is given in Section 11.4.4.

11.4.1 HIGH-LEVEL EQUATIONS FOR CALCULATING DIFFERENCED ONE-WAY LIGHT TIMES

It will be seen in Section 13 that the precision one-way light time $\hat{\rho}_1$ which is differenced and then used to calculate the computed values of F_1 and one-way *INS* and *IWS* observables is defined to be:

$$\hat{\rho}_1 = t_3(ST)_R - t_2(TAI) \quad \text{s} \quad (11-8)$$

where $t_3(ST)_R$ is the reception time in station time ST at the receiving electronics at the receiver (a receiving station on Earth or an Earth satellite) and $t_2(TAI)$ is the transmission time in International Atomic Time TAI at the spacecraft. Note that the atomic clock that reads TAI on board the spacecraft agreed with TAI on Earth prior to launching the spacecraft. This is discussed further in Section 11.4.2. In order to calculate $\hat{\rho}_1$, an expression is required for the time difference $(ET - TAI)_{t_2}$ at the transmission time t_2 at the spacecraft. Section 2 gives expressions for calculating $ET - TAI$ at a tracking station on Earth and at an Earth satellite. However, we do not have an expression for calculating $ET - TAI$ at a spacecraft on an arbitrary trajectory through the Solar System. Hence, instead of calculating the precision one-way light time $\hat{\rho}_1$, we will calculate the precision one-way light time ρ_1 which is defined to be:

$$\rho_1 = t_3(ST)_R - t_2(ET) \quad \text{s} \quad (11-9)$$

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where $t_2(\text{ET})$ is the transmission time in coordinate time ET at the spacecraft. The relation between $\hat{\rho}_1$ and ρ_1 is:

$$\hat{\rho}_1 = \rho_1 + (\text{ET} - \text{TAI})_{t_2} \quad \text{s} \quad (11-10)$$

Section 10.2.3.1.1 describes the differenced one-way light times which are used to calculate the computed values of F_1 and one-way *INS* and *IWS* observables. However, the light-time differences are differences of $\hat{\rho}_1$ defined by Eq. (11-8) instead of ρ_1 defined by Eq. (11-9) as stated in Section 10.2.3.1.1.

From Eq. (11-10), the differenced one-way light time used to calculate the computed value of an F_1 observable and each F_1 observable differenced to give the computed value of a one-way *INS* observable are given by:

$$\hat{\rho}_{1_e} - \hat{\rho}_{1_s} = \rho_{1_e} - \rho_{1_s} + \Delta \quad \text{s} \quad (11-11)$$

where

$$\Delta = (\text{ET} - \text{TAI})_{t_{2_e}} - (\text{ET} - \text{TAI})_{t_{2_s}} \quad \text{s} \quad (11-12)$$

In Eq. (11-11), the one-way light times with subscripts e and s have reception times $t_3(\text{ST})_R$ equal to the end and start of the doppler count interval T_c at the receiver. In Eq. (11-12), the time differences $(\text{ET} - \text{TAI})$ at the spacecraft are evaluated at the end and start of the transmission interval T_c' , which corresponds to the reception interval T_c at the receiver.

In order to calculate the computed value of a one-way *IWS* observable, Eqs. (11-11) and (11-12) can be used with subscripts e and s changed to Receiver 2 and Receiver 1, respectively. In these modified equations, the precision one-way light times for receivers 2 and 1 have a common reception time $t_3(\text{ST})_R$, which is equal to the data time tag. The transmission times at the spacecraft for each of the two receivers will differ by less than the Earth's radius divided by the speed of light, or 0.02 s.

The high-level equations for calculating Δ defined by Eq. (11–12) are given in the next section.

11.4.2 HIGH-LEVEL EQUATIONS FOR CALCULATING Δ

The quantity Δ , defined by Eq. (11–12), can be expressed as:

$$\Delta = \int_{t_{2s}(ET)}^{t_{2e}(ET)} I \, dET \quad \text{s} \quad (11-13)$$

where $t_{2e}(ET)$ and $t_{2s}(ET)$ are epochs at the end and start of the transmission interval at the spacecraft, and:

$$I = 1 - \frac{dT_{AI}}{dET} \quad (11-14)$$

The quantity dT_{AI} is an interval of atomic time recorded on the TAI clock carried by the spacecraft. The corresponding interval of coordinate time ET is dET . From Eq. (2–20), the quantity I is given by:

$$I = \frac{1}{c^2} \left(U + \frac{1}{2} v^2 \right) - L \quad (11-15)$$

where U is the gravitational potential at the spacecraft and v is the Solar-System barycentric velocity of the spacecraft. The constant L is defined by Eq. (2–22), evaluated at mean sea level on Earth. This initial condition is used because if the spacecraft atomic clock were placed on the surface of the Earth at mean sea level, it would agree with International Atomic Time TAI on Earth (see Eqs. (2–20) and (2–22)). The constant L is obtained by evaluating Eq. (4–12); the resulting numerical value is given by Eq. (4–13). The derivative of I with respect to coordinate time ET is given by:

$$\dot{I} = \frac{1}{c^2} \left[\dot{U} + \frac{1}{2} (v^2) \right] \quad \text{1/s} \quad (11-16)$$

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where \dot{U} and $(v^2)'$ are time derivatives of U and v^2 , respectively.

In the local geocentric space-time frame of reference, the gravitational potential U in Eq. (11–15) is due to the Earth, and v is the geocentric space-fixed velocity of the spacecraft. The constant L in the geocentric frame of reference is obtained by evaluating Eq. (4–14); the resulting numerical value is given by Eq. (4–15).

If we represent I as a cubic function of coordinate time ET in Eq. (11–13), it can be shown that the function Δ is given by:

$$\Delta = \frac{1}{2}(I_e + I_s)T - \frac{1}{12}(\dot{I}_e - \dot{I}_s)T^2 \quad \text{s} \quad (11-17)$$

where

$$T = t_{2_e}(\text{ET}) - t_{2_s}(\text{ET}) \quad \text{s} \quad (11-18)$$

In Eq. (11–17), I_e and \dot{I}_e are I and \dot{I} given by Eqs. (11–15) and (11–16), evaluated at the epoch $t_{2_e}(\text{ET})$. Similarly, I_s and \dot{I}_s are evaluated at the epoch $t_{2_s}(\text{ET})$.

The next section gives the algorithm for calculating U , \dot{U} , v^2 , and $(v^2)'$. Evaluating this algorithm at $t_{2_e}(\text{ET})$ and $t_{2_s}(\text{ET})$ and substituting the calculated quantities into Eqs. (11–15) and (11–16) gives the required values of I_e , I_s , \dot{I}_e , and \dot{I}_s , which are used to calculate Δ from Eqs. (11–17) and (11–18).

Eqs. (11–13) to (11–18) can be used in calculating the computed values of F_1 and one-way *INS* observables. However, for one-way *IWS* observables, the notation must be changed. The epoch $t_{2_e}(\text{ET})$ must be changed to $t_{2(\text{ET})_{\text{Receiver } 2}}$, the transmission time at the spacecraft for receiver 2. The epoch $t_{2_s}(\text{ET})$ must be changed to $t_{2(\text{ET})_{\text{Receiver } 1}}$, the transmission time at the spacecraft for receiver 1. In Eq. (11–17), the subscripts e and s refer to the epochs $t_{2(\text{ET})_{\text{Receiver } 2}}$ and $t_{2(\text{ET})_{\text{Receiver } 1}}$, respectively.

11.4.3 ALGORITHM FOR CALCULATING THE ARGUMENTS U , \dot{U} , v^2 , AND $(v^2)'$ OF I AND \dot{I}

The gravitational potential U at the spacecraft and its time derivative \dot{U} are calculated from:

$$U = U_{\text{pm}} + U_{\text{obl}} \quad \text{km}^2/\text{s}^2 \quad (11-19)$$

$$\dot{U} = \dot{U}_{\text{pm}} + \dot{U}_{\text{obl}} \quad \text{km}^2/\text{s}^3 \quad (11-20)$$

where U_{pm} is the potential at the spacecraft due to bodies treated as point masses. The term U_{obl} is the additional potential at the spacecraft due to the oblateness of a nearby body. The algorithms for calculating U_{pm} and U_{obl} and their time derivatives are given in Subsections 11.4.3.1 and 11.4.3.2.

The equations for calculating the terms v^2 and $(v^2)'$ of Eqs. (11-15) and (11-16) are given in Subsection 11.4.3.3.

All quantities calculated in Subsections 11.4.3.1 to 11.4.3.3 are evaluated at the transmission time t_2 of the one-way spacecraft light-time solution.

The algorithms for calculating U , \dot{U} , v^2 , and $(v^2)'$ apply in general in the Solar-System barycentric space-time frame of reference. The simplifications that apply when calculating these quantities in the local geocentric space-time frame of reference are noted.

11.4.3.1 Gravitational Potential at the Spacecraft Due to Point-Mass Bodies

1. Obtain the Solar-System barycentric (C) space-fixed position and velocity vectors of bodies k consisting of the Sun, Mercury, Venus, the Earth, the Moon, the barycenters of the planetary systems Mars through Pluto, and possibly one or more asteroids or comets. These vectors are available from Steps 7 and 17 of the spacecraft light-time solution (Section 8.3.6).

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$$\mathbf{r}_k^C, \dot{\mathbf{r}}_k^C \quad (11-21)$$

2. If the spacecraft is free and is within the sphere of influence of one of the outer planet systems Mars through Pluto, or if the spacecraft is landed and the lander body is the planet or one of the satellites of one of these outer planet systems, obtain the space-fixed position and velocity vectors of bodies k consisting of the planet and each satellite on the satellite ephemeris relative to the barycenter P of the planetary system:

$$\mathbf{r}_k^P, \dot{\mathbf{r}}_k^P \quad (11-22)$$

These vectors are available from Steps 8 and 17 of the spacecraft light-time solution.

3. Add the vectors (11-21) for k = the planetary system P to the vectors (11-22) to give the Solar-System barycentric position and velocity vectors of the planet and each satellite on the satellite ephemeris.
4. Obtain the space-fixed Solar-System barycentric position, velocity, and acceleration vectors of the free or landed spacecraft p :

$$\mathbf{r}_p^C, \dot{\mathbf{r}}_p^C, \ddot{\mathbf{r}}_p^C \quad (11-23)$$

These vectors are calculated in Steps 7 to 11, 17, and 18 of the spacecraft light-time solution.

5. Given the Solar-System barycentric position and velocity vectors of the Sun, the Moon, the planets, the planetary satellites, and possibly one or more asteroids or comets calculated in Steps 1 to 3 (bodies k) and the spacecraft (p) in Step 4, calculate the space-fixed position and velocity vectors of the spacecraft relative to each body k :

$$\mathbf{r}_p^k = \mathbf{r}_p^C - \mathbf{r}_k^C \quad \text{km} \quad (11-24)$$

$$\dot{\mathbf{r}}_p^k = \dot{\mathbf{r}}_p^C - \dot{\mathbf{r}}_k^C \quad \text{km/s} \quad (11-25)$$

6. Calculate the range and the range rate from each body k to the spacecraft p :

$$r_{kp} = |\mathbf{r}_p^k| \quad \text{km} \quad (11-26)$$

$$\dot{r}_{kp} = \frac{\mathbf{r}_p^k}{r_{kp}} \cdot \dot{\mathbf{r}}_p^k \quad \text{km/s} \quad (11-27)$$

7. The gravitational constants μ_k for the bodies k in units of km^3/s^2 are obtained from the planetary, small-body, and satellite ephemerides as described in Sections 3.1.2.2 and 3.2.2.1.
8. Calculate the point-mass gravitational potential U_{pm} and its time derivative \dot{U}_{pm} from:

$$U_{pm} = \sum_k \frac{\mu_k}{r_{kp}} \quad \text{km}^2/\text{s}^2 \quad (11-28)$$

$$\dot{U}_{pm} = - \sum_k \frac{\mu_k}{r_{kp}^2} \dot{r}_{kp} \quad \text{km}^2/\text{s}^3 \quad (11-29)$$

In the local geocentric space-time frame of reference, the summations over bodies k include one body only, namely, the Earth. The space-fixed position, velocity, and acceleration vectors of the spacecraft relative to the Earth are obtained by interpolating the spacecraft ephemeris in Steps 9 and 17 of the spacecraft light-time solution. Substituting these vectors into Eqs. (11-26) and (11-27) gives r_{kp} and \dot{r}_{kp} .

11.4.3.2 Gravitational Potential at the Spacecraft Due to a Nearby Oblate Body

The term U_{obl} of Eq. (11–19) is the gravitational potential at the spacecraft due to the oblateness of a nearby planet. If the spacecraft is within the sphere of influence of Mercury, Venus, or the Earth, U_{obl} is calculated for that planet. If the spacecraft is within the sphere of influence of one of the outer planet systems Mars through Pluto, U_{obl} is calculated for the planet of that system. The oblateness potential is not calculated for the Sun, the Moon, satellites of the outer planet systems, asteroids, or comets. Note that if the spacecraft is landed on a planet or a planetary satellite, it will be within the sphere of influence of the planet and hence U_{obl} due to the planet will be calculated.

1. Step 5 of Section 11.4.3.1 gives the space-fixed position and velocity vectors of the spacecraft (p) relative to the nearby oblate planet (k):

$$\mathbf{r}_p^k, \dot{\mathbf{r}}_p^k \quad (11-30)$$

2. Substituting these vectors into Eqs. (11–26) and (11–27) gives the range and range-rate from the oblate planet to the spacecraft:

$$r_{kp}, \dot{r}_{kp} \quad (11-31)$$

3. The space-fixed unit vector \mathbf{P} directed toward the oblate planet's north pole (axis of rotation) of date is calculated from:

$$\mathbf{P} = \begin{bmatrix} \cos \delta \cos \alpha \\ \cos \delta \sin \alpha \\ \sin \delta \end{bmatrix} \quad (11-32)$$

where α and δ are the right ascension and declination of the planet's north pole of date relative to the mean Earth equator and equinox of J2000. For each planet except the Earth, α and δ are calculated from Eqs. (6–8), (6–9), and (5–65). The coefficients in these linear equations

are obtained from Table I of Davies *et al.* (1996). For the planet Neptune, α and δ should be supplemented with the nutation terms $\Delta\alpha$ and $\Delta\delta$, which are calculated from Eqs. (6–15), (6–17), and (5–65). The coefficients in these equations are obtained from Table I of Davies *et al.* (1996). For the Earth, α and δ are calculated from Eqs. (5–142), (5–143), and (5–65).

4. The gravitational potential at the spacecraft due to the oblateness of a nearby planet is a function of the latitude ϕ of the spacecraft relative to the planet's equator. Given the quantities (11–30) to (11–32), the sine of the latitude ϕ and its time derivative are calculated from:

$$\sin \phi = \mathbf{P} \cdot \frac{\mathbf{r}_p^k}{r_{kp}} \quad (11-33)$$

$$(\sin \phi)' = \frac{\mathbf{P}}{r_{kp}} \cdot \left(\dot{\mathbf{r}}_p^k - \frac{\dot{r}_{kp}}{r_{kp}} \mathbf{r}_p^k \right) \quad (11-34)$$

5. Given $r = r_{kp}$ and $\dot{r} = \dot{r}_{kp}$ from Step 2, and the gravitational constant of the planet $\mu = \mu_k$ from Step 7 of Section 11.4.3.1, the gravitational potential at the spacecraft due to the zonal harmonic coefficients of a nearby planet and the time derivative of the gravitational potential are calculated from:

$$U_{\text{obl}} = -\frac{\mu}{r} \sum_{n=2}^N J_n \left(\frac{a}{r} \right)^n P_n \quad (11-35)$$

$$\dot{U}_{\text{obl}} = \frac{\mu}{r} \sum_{n=2}^N J_n \left(\frac{a}{r} \right)^n \left[(n+1) \frac{\dot{r}}{r} P_n - (P_n') (\sin \phi)' \right] \quad (11-36)$$

where

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- J_n = zonal harmonic coefficient of degree n
 N = highest degree n of zonal harmonics (obtained from GIN file) or 8, whichever is smaller
 a = mean equatorial radius of planet
 P_n = Legendre polynomial of degree n in $\sin \phi$
 P_n' = derivative of P_n with respect to $\sin \phi$

The Legendre polynomial P_n is computed recursively from Eqs. (175) to (177) of Moyer (1971). The quantity P_n' is computed recursively from Eqs. (178) and (179) of Moyer (1971).

11.4.3.3 Square of Spacecraft Velocity

1. Step 4 of Section 11.4.3.1 gives the space-fixed velocity and acceleration vectors of the free or landed spacecraft (p) relative to the Solar-System barycenter (C):

$$\dot{\mathbf{r}}_p^C, \ddot{\mathbf{r}}_p^C \quad (11-37)$$

In the local geocentric space-time frame of reference, these vectors are referred to the Earth (superscript E instead of C). They are obtained as described in Step 8 of Section 11.4.3.1.

2. In the Solar-System barycentric space-time frame of reference, the square of the Solar-System barycentric velocity v of the spacecraft is calculated from:

$$v^2 = \dot{\mathbf{r}}_p^C \cdot \dot{\mathbf{r}}_p^C \quad (11-38)$$

The time derivative of v^2 is calculated from:

$$\left(v^2\right)' = 2 \dot{\mathbf{r}}_p^C \cdot \ddot{\mathbf{r}}_p^C \quad (11-39)$$

In the local geocentric space-time frame of reference, the square of the geocentric space-fixed velocity v of the spacecraft and its time derivative are calculated from these same equations with the superscript C changed to E.

11.4.4 CALCULATION OF PRECISION ONE-WAY LIGHT TIME ρ_1

The computed values of one-way doppler (F_1) observables, one-way narrowband spacecraft interferometry (INS) observables, and one-way wideband spacecraft interferometry (IWS) observables are calculated from differenced one-way range $\hat{\rho}_1$ calculated from Eq. (11-11). The right-hand side of this equation contains differenced one-way range ρ_1 , where each of the two one-way ranges ρ_1 is defined by Eq. (11-9). Substituting Eq. (11-1) into Eq. (11-9) gives:

$$\rho_1 = t_3(\text{ST}) - t_2(\text{ET}) + \tau_D \quad \text{s} \quad (11-40)$$

where $t_3(\text{ST})$ is the reception time at the tracking point of the receiver (defined in the fifth paragraph of Section 11.2) and τ_D is the down-leg delay at the receiver.

The precision one-way light time ρ_1 defined by Eq. (11-9) or (11-40) is calculated as the following sum of terms:

$$\begin{aligned} \rho_1 = & \frac{r_{23}}{c} + RLT_{23} \\ & - (\text{ET} - \text{TAI})_{t_3} \\ & - (\text{TAI} - \text{UTC})_{t_3} \\ & - (\text{UTC} - \text{ST})_{t_3} \quad \text{s} \quad (11-41) \\ & + \frac{1}{10^3 c} [\Delta_A \rho(t_3) + \Delta_{SC} \rho_{23}] \\ & + \tau_D \end{aligned}$$

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where c is the speed of light in kilometers per second. This equation was obtained from Eq. (11-7) for the precision round-trip light time ρ by deleting all up-leg terms, the time differences at t_1 , the up-leg delay τ_U at the transmitter, and the round-trip range bias R_c .

The surviving terms in Eq. (11-41) are obtained from the spacecraft light-time solution or are calculated as described in Section 11.3.2. In this case, however, the spacecraft light-time solution is one-way, not round-trip.

Equation (11-41) does not include corrections due to the troposphere or due to charged particles. These corrections are calculated in the Regres editor and are included in Eqs. (10-24) to (10-26) for the corrections $\Delta\rho_{1e}$, $\Delta\rho_{1s}$, and $\Delta\rho_1$ to ρ_1 given by Eq. (11-41). These corrections to ρ_1 are handled separately as described in Sections 10.1 and 10.2.

Eq. (11-41) accounts for the location of the tracking point of the receiver (see Section 11.3.2). However, unless the spacecraft is a GPS satellite, the phase center of the spacecraft is currently assumed to be located at the center of mass of the spacecraft (see the spacecraft light-time solution, Section 8.3.6, Step 9). This affects ρ_1 calculated from Eq. (11-41) and ρ calculated from Eq. (11-7).

11.5 PRECISION ONE-WAY LIGHT TIME ρ_1 FOR GPS/TOPEX OBSERVABLES

This section gives the formulation for calculating the precision one-way light time ρ_1 (in units of kilometers), which is the computed value of a GPS/TOPEX pseudo-range or carrier-phase observable. For these observables, the transmitter is a GPS Earth satellite, and the receiver is either a TOPEX Earth satellite (or equivalent) or a GPS receiving station on Earth.

The definition of the precision one-way light time ρ_1 (in units of kilometers) is given in Section 11.5.1. Section 11.5.2 gives the equation for calculating ρ_1 as a sum of terms. One of the terms of the equation for ρ_1 contains the geometrical phase correction $\Delta\Phi$, which is only calculated for carrier-phase observables. The formulation for calculating $\Delta\Phi$ is given in Section 11.5.3. The

equation for ρ_1 contains terms for the variable parts of the phase center offsets at the transmitting GPS satellite and the receiving TOPEX satellite or the GPS receiving station on Earth. These variable phase-center offsets are calculated for carrier-phase observables only as described in Section 11.5.4.

11.5.1 DEFINITION OF ρ_1

The definition of the precision one-way light time ρ_1 (in units of kilometers), which is the computed value of a GPS/TOPEX pseudo-range or carrier-phase observable, is given by:

$$\rho_1 = c [t_3(\text{ST})_{\text{R}} - t_2(\text{ST})] \quad \text{km} \quad (11-42)$$

where c is the speed of light in kilometers per second. Substituting Eq. (11-1) into Eq. (11-42) gives the following alternate definition of ρ_1 :

$$\rho_1 = c [t_3(\text{ST}) - t_2(\text{ST}) + \tau_{\text{D}}] \quad \text{km} \quad (11-43)$$

In these equations, $t_2(\text{ST})$ is the transmission time in station time ST at the tracking point of the GPS satellite. The reception time in station time ST at the tracking point of the receiving TOPEX satellite or the GPS receiving station on Earth is $t_3(\text{ST})$. Adding the down-leg delay τ_{D} to $t_3(\text{ST})$ gives the reception time $t_3(\text{ST})_{\text{R}}$ at the receiving electronics. If the receiver is the TOPEX satellite, τ_{D} is set to zero.

Observed values of GPS/TOPEX pseudo-range and carrier-phase observables are obtained with an L1-band transmitter frequency and also with an L2-band transmitter frequency. The values of these two transmitter frequencies are given in Eq. (7-1). Each observable pair is used to construct a weighted average observable, which is free of the effects of charged particles. The weighting equations are Eqs. (7-2) to (7-4). In principal, each computed observable should be computed using an L1-band transmitter frequency and also using an L2-band transmitter frequency. A weighted average computed observable is then computed using Eqs. (7-2) to (7-4). The following section

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gives the equation for the computed value of a GPS/TOPEX pseudo-range or carrier-phase observable. Each frequency-dependent term must be computed as a weighted average using Eqs. (7–2) to (7–4). The remaining terms are computed once. The frequency-dependent terms are the constant and variable phase-center offsets at the transmitter and the receiver and the geometrical phase correction for carrier-phase observables.

11.5.2 CALCULATION OF ρ_1

The precision one-way light time ρ_1 defined by Eq. (11–42) or (11–43) is calculated as the following sum of terms:

$$\begin{aligned} \rho_1 = c \left[\frac{r_{23}}{c} + RLT_{23} \right. \\ - (ET - TAI)_{t_3} + (ET - TAI)_{t_2} \\ - (TAI - MT)_{t_3} + (TAI - GPS)_{t_2} \\ - (MT - ST)_{t_3} + (GPS - ST)_{t_2} \\ \left. + Bias + \frac{\Delta\Phi}{2\pi f} + \frac{\Delta_A \rho(t_3)}{f} + \frac{\Delta_A \rho(t_2)}{f} + \tau_D \right] \quad \text{km} \quad (11-44) \end{aligned}$$

The down-leg range r_{23} , the down-leg relativistic light-time delay RLT_{23} , the three time differences at the reception time t_3 , and the three time differences at the transmission time t_2 are all calculated in the down-leg spacecraft light-time solution as specified in Section 8.3.6.

In Eq. (11–44), the parameter MT in the time differences at the reception time t_3 is master time at the receiver. If the receiver is a GPS receiving station on Earth, MT is GPS master time (denoted as GPS). If the receiver is the TOPEX satellite, MT is TOPEX master time (denoted as TPX). Note that GPS master time is also used in the time differences at the transmission time t_2 at a GPS satellite.

The parameter *Bias* is a solve-for bias in seconds. One estimate of the parameter *Bias* is obtained by fitting to pseudo-range observables, and a second independent estimate of the parameter *Bias* is obtained by fitting to carrier-phase observables.

The initial value of a carrier-phase observable will be determined modulo one cycle (it will be continuous thereafter), which will differ drastically from the computed value of the carrier-phase observable. Hence, it is necessary to include an estimable bias in the computed value of carrier-phase observables. This bias must be different for each receiver/transmitter pair. In practice, the bias for carrier-phase observables and the independent bias for pseudo-range observables are specified by receiving station (a GPS receiving station on Earth or the TOPEX satellite) in time blocks. For each receiver, a new time block is used for each separate pass of data and each time the transmitter changes.

The initial value of a carrier-phase observable will be adjusted in the data editor so that it is approximately equal to the corresponding pseudo-range observable. This will result in much smaller estimated biases for carrier-phase observables.

The geometrical phase correction $\Delta\Phi$ is the lag in the measured phase at the receiver (in radians) due to the rotation of the receiver relative to the transmitter. It is calculated for carrier-phase observables only from the formulation given in Section 11.5.3.

The down-leg range r_{23} is the distance from the nominal phase center of the transmitting GPS satellite at the transmission time t_2 to the nominal phase center of the receiving TOPEX satellite or a GPS receiving station on Earth at the reception time t_3 . The terms $\Delta_A\rho(t_3)$ and $\Delta_A\rho(t_2)$ in cycles divided by the down-leg carrier frequency f in cycles per second are changes in the down-leg light time r_{23} / c due to transmission and reception at the actual phase centers instead of the nominal phase centers. Positive and negative values of the variable phase-center offsets $\Delta_A\rho(t_3)$ and $\Delta_A\rho(t_2)$ correspond to increases and decreases in the down-leg range and light time. The variable phase-center offsets

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are calculated from the formulation of Section 11.5.4. They are calculated for carrier-phase observables only.

The down-leg delay τ_D at the receiver is obtained from the record of the OD file for the data point. However, if the receiver is the TOPEX satellite, it will probably be set to zero.

Eq. (11-44) does not include corrections for the troposphere or for charged particles. Pseudo-range and carrier-phase observables are calculated as a weighted average, which eliminates the effects of charged particles. Troposphere corrections are not calculated if the receiver is the TOPEX satellite. If the receiver is a GPS receiving station on Earth, troposphere corrections are calculated in the Regres editor and are placed in the first term of Eq. (10-26) multiplied by the speed of light c , which gives the correction $\Delta\rho_1$ in kilometers to ρ_1 given by Eq. (11-44). This correction to ρ_1 is handled separately as described in Sections 10.1 and 10.2.

11.5.3 FORMULATION FOR CALCULATING THE GEOMETRICAL PHASE CORRECTION $\Delta\Phi$

The geometrical phase correction $\Delta\Phi$ (in radians) in Eq. (11-44) is only calculated for GPS/TOPEX carrier-phase observables. It is the lag in the measured phase of the received signal at the receiver due to the rotation of the receiver relative to the transmitter. It will be seen in Section 13 that carrier-phase observables are proportional to the phase of a reference signal minus the phase of the received signal at the TOPEX satellite or at a GPS receiving station on Earth. Since the phase of the received signal is the phase of the transmitted signal minus the phase lag $\Delta\Phi$, the sign of the term in Eq. (11-44) which contains the phase lag $\Delta\Phi$ is positive.

The formulation for calculating the geometrical phase correction was obtained from Wu *et al.* (1990). It applies for a right-circularly-polarized wave propagated from the transmitter to the receiver. Section 11.5.3.1 gives the algorithm for calculating the geometrical phase correction $\Delta\Phi$ in radians. Section 11.5.3.2 describes the calculation of the space-fixed unit vectors along the axes of

the transmitting GPS satellite and the receiving TOPEX satellite. Section 11.5.3.3 describes the calculation of the space-fixed unit vectors along the north, east, and zenith vectors at a GPS receiving station on Earth. Calculation of the unit vector \mathbf{k} from the transmitter to the receiver is described in Section 11.5.3.4. Section 11.5.3.5 describes how the frequency-dependent geometrical phase correction $\Delta\Phi/2\pi f$ in Eq. (11–44) is calculated as a weighted average, as discussed in Section 11.5.1.

11.5.3.1 Algorithm for Computing the Geometrical Phase Correction

From Eq. (20) of Wu *et al.* (1990), the effective dipole \mathbf{D} for the receiving antenna at the TOPEX satellite or at a GPS receiving station on Earth is given by:

$$\mathbf{D} = \mathbf{x} - \mathbf{k} (\mathbf{k} \cdot \mathbf{x}) + \mathbf{k} \times \mathbf{y} \quad (11-45)$$

where \mathbf{x} and \mathbf{y} are space-fixed unit vectors along the x and y axes of the receiving antenna (see Wu *et al.* (1990), Figure 1) at the reception time t_3 and \mathbf{k} is a space-fixed unit vector directed from the transmitting GPS satellite at the transmission time t_2 to the receiver at the reception time t_3 . The effective dipole \mathbf{D}' for the transmitting antenna at the GPS satellite is given by Eq. (28) of Wu *et al.* (1990):

$$\mathbf{D}' = \mathbf{x}' - \mathbf{k} (\mathbf{k} \cdot \mathbf{x}') - \mathbf{k} \times \mathbf{y}' \quad (11-46)$$

where \mathbf{x}' and \mathbf{y}' are space-fixed unit vectors along the x' and y' axes of the transmitting antenna (see Wu *et al.* (1990), Figure 1) at the transmission time t_2 . Calculation of the unit vectors in Eqs. (11–45) and (11–46) is described in the following three sections.

Unit vectors along the effective dipoles \mathbf{D} and \mathbf{D}' are calculated from:

$$\hat{\mathbf{D}} = \frac{\mathbf{D}}{D} \quad (11-47)$$

and

$$\hat{\mathbf{D}}' = \frac{\mathbf{D}'}{D'} \quad (11-48)$$

where D is the magnitude of \mathbf{D} and D' is the magnitude of \mathbf{D}' . The unit vectors $\hat{\mathbf{D}}$ and $\hat{\mathbf{D}}'$ are normal to \mathbf{k} .

The phase lag $\Delta\phi$ is a discontinuous function of time, which will be converted below to the continuous function of time $\Delta\Phi$. The discontinuous phase lag $\Delta\phi$ is plus or minus the angle between $\hat{\mathbf{D}}$ and $\hat{\mathbf{D}}'$. It is calculated from Eqs. (30) and (31) of Wu *et al.* (1990):

$$\Delta\phi = \text{sign}(\zeta) \cos^{-1}(\hat{\mathbf{D}}' \cdot \hat{\mathbf{D}}) \quad \text{rad} \quad (11-49)$$

where

$$\zeta = \mathbf{k} \cdot (\hat{\mathbf{D}}' \times \hat{\mathbf{D}}) \quad (11-50)$$

In Eq. (11-49), the arccosine function gives an angle in the range of 0 to π radians. Adding the sign function to this equation gives $\Delta\phi$ calculated from Eqs. (11-49) and (11-50) which has a range of $-\pi$ to π radians. As $\Delta\phi$ increases slowly through π radians, it drops by 2π . Similarly, when $\Delta\phi$ decreases through $-\pi$ radians, it jumps by 2π . The discontinuous phase lag $\Delta\phi$ calculated from Eqs. (11-49) and (11-50) is converted to the continuous phase lag $\Delta\Phi$ using Eqs. (29) and (32) of Wu *et al.* (1990):

$$\Delta\Phi = 2\pi N + \Delta\phi \quad \text{rad} \quad (11-51)$$

where

$$N = \text{nint} \left[\frac{\Delta\Phi_{\text{prev}} - \Delta\phi}{2\pi} \right] \quad (11-52)$$

where nint is the nearest integer function and $\Delta\Phi_{\text{prev}}$ is the previously computed value of the continuous phase lag $\Delta\Phi$. The phase lag $\Delta\Phi$ must be computed separately for each pass of each transmitter/receiver pair. The value of N should

be set to zero at the beginning of each pass. Each time $\Delta\phi$ suffers a discontinuity of $\pm 2\pi$, the integer N will change by minus or plus 1. Note that the nearest integer function nint will only give the correct value of N if the change in the continuous angle $\Delta\Phi$ is less than 180° . It is assumed that the data spacing for carrier-phase observables will be small enough so that this will be the case.

11.5.3.2 Unit Vectors \mathbf{x}' and \mathbf{y}' at the Transmitting GPS Satellite and Unit Vectors \mathbf{x} and \mathbf{y} at the Receiving TOPEX Satellite

The space-fixed unit vectors \mathbf{X} , \mathbf{Y} , and \mathbf{Z} are aligned with the x , y , and z axes of the spacecraft-fixed coordinate system for the TOPEX satellite at the reception time t_3 and for a GPS satellite at the transmission time t_2 . The \mathbf{X} , \mathbf{Y} , and \mathbf{Z} vectors for the TOPEX satellite are obtained when interpolating the PV file for the TOPEX satellite in Step 3 of the algorithm given in Section 7.3.3, which is evaluated in Step 2 of the spacecraft light-time solution (Section 8.3.6). The \mathbf{X} , \mathbf{Y} , and \mathbf{Z} vectors for the GPS satellite are obtained when interpolating the PV file for the GPS satellite in Step 3 of the algorithm given in Section 7.3.3, which is evaluated in Step 9 of the spacecraft light-time solution.

The relation between the unit vectors \mathbf{X} , \mathbf{Y} , and \mathbf{Z} interpolated from the PV files for the GPS and TOPEX satellites and the unit vectors \mathbf{x}' , \mathbf{y}' , and \mathbf{z}' for the transmitting GPS satellite and \mathbf{x} , \mathbf{y} , and \mathbf{z} for the receiving TOPEX satellite, which are required to compute the effective dipoles \mathbf{D} and \mathbf{D}' from Eqs. (11–45) and (11–46), must be determined.

The \mathbf{X} - \mathbf{Y} and \mathbf{x} - \mathbf{y} planes at the TOPEX satellite are the same plane. It is the antenna plane which is perpendicular to the boresight vector \mathbf{z} . However, $\mathbf{x} \times \mathbf{y} = \mathbf{z}$ which is nominally directed up and $\mathbf{X} \times \mathbf{Y} = \mathbf{Z} = -\mathbf{z}$ which is nominally directed down. Given \mathbf{X} , \mathbf{Y} , and \mathbf{Z} for the TOPEX satellite, an \mathbf{x} - \mathbf{y} - \mathbf{z} system can be constructed as follows:

$$\begin{aligned} \mathbf{x} &= \mathbf{Y} \\ \mathbf{y} &= \mathbf{X} \\ \mathbf{z} &= -\mathbf{Z} \quad (\text{not used}) \end{aligned} \tag{11-53}$$

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The alignment of \mathbf{x} with the \mathbf{Y} spacecraft axis is arbitrary. The actual orientation of \mathbf{x} in the \mathbf{X} - \mathbf{Y} plane is unknown. The use of \mathbf{x} computed from Eq. (11-53) will produce a constant error in the phase lag computed from Eqs. (11-45) to (11-52).

The \mathbf{X} - \mathbf{Y} and \mathbf{x}' - \mathbf{y}' planes at the transmitting GPS satellite are the same plane. It is the antenna plane which is perpendicular to the boresight vector \mathbf{z}' . Also, $\mathbf{x}' \times \mathbf{y}' = \mathbf{z}'$ and $\mathbf{X} \times \mathbf{Y} = \mathbf{Z}$ where $\mathbf{Z} = \mathbf{z}'$ is nominally directed down. Given \mathbf{X} , \mathbf{Y} , and \mathbf{Z} for the transmitting GPS satellite, an \mathbf{x}' - \mathbf{y}' - \mathbf{z}' system can be constructed as follows:

$$\begin{aligned}\mathbf{x}' &= \mathbf{X} \\ \mathbf{y}' &= \mathbf{Y} \\ \mathbf{z}' &= \mathbf{Z} \quad (\text{not used})\end{aligned}\tag{11-54}$$

The alignment of \mathbf{x}' with the \mathbf{X} spacecraft axis is arbitrary. The actual orientation of \mathbf{x}' in the \mathbf{X} - \mathbf{Y} plane is unknown. The use of \mathbf{x}' computed from Eq. (11-54) will produce a constant error in the phase lag computed from Eqs. (11-45) to (11-52).

The constant error in the computed phase lag $\Delta\Phi$ will be absorbed into the estimated value of the carrier-phase bias *Bias* in Eq. (11-44).

11.5.3.3 Unit Vectors \mathbf{x} and \mathbf{y} at a GPS Receiving Station on Earth

The north \mathbf{N} , east \mathbf{E} , and zenith \mathbf{Z} unit vectors at the reception time t_3 at a GPS receiving station on Earth are calculated during the calculation of the Earth-fixed position vector of the tracking station (using the formulation of Section 5) and during the calculation of auxiliary angles at the tracking station (using the formulation of Section 9). These unit vectors are calculated in the Earth-fixed coordinate system and have rectangular components referred to the true pole, prime meridian, and equator of date. The \mathbf{N} , \mathbf{E} , and \mathbf{Z} unit vectors can be transformed from the Earth-fixed coordinate system to the space-fixed coordinate system (rectangular components referred to the mean Earth equator and equinox of J2000) using:

$$\mathbf{N}_{\text{SF}} = T_{\text{E}}(t_3) \mathbf{N} \quad \mathbf{N} \rightarrow \mathbf{E}, \mathbf{Z} \quad (11-55)$$

where the subscript SF refers to space-fixed components of the vector. The Earth-fixed to space-fixed transformation matrix $T_{\text{E}}(t_3)$ at the reception time t_3 at the GPS receiving station on Earth is calculated from the formulation of Section 5.3. It is available from Step 2 of the spacecraft light-time solution (Section 8.3.6).

The \mathbf{N} and \mathbf{E} vectors are in the antenna plane (normal to the boresight vector \mathbf{Z}) of the GPS receiving station on Earth. Given the \mathbf{N}_{SF} , \mathbf{E}_{SF} , and \mathbf{Z}_{SF} unit vectors computed from Eq. (11-55), with rectangular components referred to the mean Earth equator and equinox of J2000, the required unit vectors \mathbf{x} , \mathbf{y} , and \mathbf{z} of the receiving antenna (which are used to calculate the effective dipole \mathbf{D} from Eq. (11-45)) can be constructed from:

$$\begin{aligned} \mathbf{x} &= \mathbf{N}_{\text{SF}} \\ \mathbf{y} &= -\mathbf{E}_{\text{SF}} \\ \mathbf{z} &= \mathbf{Z}_{\text{SF}} \quad (\text{not used}) \end{aligned} \quad (11-56)$$

Note that $\mathbf{x} \times \mathbf{y} = \mathbf{z}$ which is directed up. The alignment of \mathbf{x} with \mathbf{N} is arbitrary. The actual orientation of \mathbf{x} in the N-E plane is unknown. The use of \mathbf{x} calculated from Eq. (11-56) will produce a constant error in the computed phase lag $\Delta\Phi$, which will be absorbed into the estimated carrier-phase bias *Bias*.

11.5.3.4 Unit Vector \mathbf{k} Along Light Path From Transmitter to Receiver

Since relativistic effects are not included in the computed phase lag $\Delta\Phi$, the unit vector \mathbf{k} used in Eqs. (11-45), (11-46), and (11-50) can be computed from:

$$\mathbf{k} = \frac{\mathbf{r}_{23}}{r_{23}} \quad (11-57)$$

where \mathbf{r}_{23}/r_{23} is the down-leg unit vector calculated in Step 14 of the spacecraft light-time solution (Section 8.3.6).

11.5.3.5 Calculating the Geometrical Phase Correction $\Delta\Phi/2\pi f$ as a Weighted Average

The geometrical phase correction $\Delta\Phi/2\pi f$ in Eq. (11-44) must be computed as a weighted average of the value at the L1-band transmitter frequency and the value at the L2-band transmitter frequency, as discussed in Section 11.5.1. The weighting equations are Eqs. (7-2) to (7-4). Substituting $\Delta\Phi/2\pi L1$ into the first term of Eq. (7-2) and $\Delta\Phi/2\pi L2$ into the second term gives the following expression for the weighted average (WA) value of the geometrical phase correction $\Delta\Phi/2\pi f$:

$$\left[\frac{\Delta\Phi}{2\pi f} \right]_{\text{WA}} = \frac{\Delta\Phi}{2\pi (L1 + L2)} \quad \text{s} \quad (11-58)$$

where $L1$ and $L2$ are given by Eq. (7-1). This value of $\Delta\Phi/2\pi f$ should be used in Eq. (11-44).

11.5.4 CALCULATION OF VARIABLE PHASE-CENTER OFFSETS

Two tables can be used to obtain the variable phase-center offset $\Delta_A\rho(t_3)$ at the reception time t_3 at the TOPEX satellite. One table gives the variable phase-center offset $\Delta_A\rho(t_3)$ in cycles for an L1-band carrier frequency and the second table gives $\Delta_A\rho(t_3)$ in cycles for an L2-band carrier frequency. Two similar tables are used for reception at a GPS receiving station on Earth. Variable phase-center offsets at the transmitting GPS satellite have not been measured, and hence the term $\Delta_A\rho(t_2)/f$ in Eq. (11-44) is zero.

Each of the above-mentioned tables gives the variable phase-center offset for a particular receiver and band. The arguments for these tables are the antenna zenith angle and the antenna azimuth angle. Section 11.5.4.1 gives the equations for converting the auxiliary azimuth and elevation angles calculated at the reception time at the TOPEX satellite and at a GPS receiving station on Earth

to the required antenna angles. The equations for interpolating the tables with these angles are given in Section 11.5.4.2. Section 11.5.4.3 gives the equation for calculating the term $\Delta_A \rho(t_3)/f$ of Eq. (11-44) as a weighted average of the L1-band and L2-band values. This term is only calculated for GPS/TOPEX carrier-phase observables.

11.5.4.1 Calculation of Angular Arguments

The arguments for the tables (which give the variable phase-center offset $\Delta_A \rho(t_3)$ in cycles at the TOPEX satellite and at a GPS receiving station on Earth) are the antenna zenith angle z_A and the antenna azimuth angle σ_A . The antenna zenith angle is measured from the antenna boresight direction, which is directed up for both receivers. The antenna azimuth angle is measured counter clockwise (when viewed from above the antenna) from the x axis of the antenna. Regres calculates auxiliary elevation γ_{GPS} and azimuth σ_{GPS} angles at the reception time t_3 at a GPS receiving station on Earth (Section 9.3.3.2). It also calculates differently defined auxiliary elevation γ_{TPX} and azimuth σ_{TPX} angles at the reception time t_3 at the TOPEX satellite (Section 9.5.1). The following equations transform the auxiliary angles to the angular arguments of the variable phase-center offset tables. For a GPS receiving station on Earth,

$$\begin{aligned} z_A &= \frac{\pi}{2} - \gamma_{\text{GPS}} & 0 \leq z_A \leq \frac{\pi}{2} \\ \sigma_A &= 2\pi - \sigma_{\text{GPS}} & 0 \leq \sigma_A \leq 2\pi \end{aligned} \quad (11-59)$$

For the TOPEX satellite,

$$\begin{aligned} z_A &= \frac{\pi}{2} + \gamma_{\text{TPX}} & 0 \leq z_A \leq \pi \\ \sigma_A &= 2\pi - \sigma_{\text{TPX}} & 0 \leq \sigma_A \leq 2\pi \end{aligned} \quad (11-60)$$

These four angles must be converted from radians to degrees.

11.5.4.2 Interpolation of Variable Phase-Center Offset Tables

The variable phase-center offset tables give values of the variable phase-center offset $\Delta_A \rho$ in cycles every 5° in the antenna zenith angle z_A and in the antenna azimuth angle σ_A . The arguments z_A and σ_A will be between the tabular values z_1 and z_2 and σ_1 and σ_2 , respectively. The value of $\Delta_A \rho$ at the interpolation point (z_A, σ_A) , which will be denoted as $\Delta_A \rho(z_A, \sigma_A)$, can be obtained by using bilinear interpolation. This requires three linear interpolations. First interpolate at σ_1 to z_A . Then interpolate at σ_2 to z_A . Finally, interpolate at z_A to σ_A . The result of these calculations is given by:

$$\begin{aligned} \Delta_A \rho(z_A, \sigma_A) = & \Delta_A \rho(z_1, \sigma_1)(1-f_z)(1-f_\sigma) \\ & + \Delta_A \rho(z_2, \sigma_1)f_z(1-f_\sigma) \\ & + \Delta_A \rho(z_1, \sigma_2)(1-f_z)f_\sigma \\ & + \Delta_A \rho(z_2, \sigma_2)f_z f_\sigma \end{aligned} \quad \text{cycles} \quad (11-61)$$

where

$$f_z = \frac{z_A - z_1}{z_2 - z_1} \quad (11-62)$$

$$f_\sigma = \frac{\sigma_A - \sigma_1}{\sigma_2 - \sigma_1} \quad (11-63)$$

11.5.4.3 Calculation of Variable Phase-Center Offset as a Weighted Average

Let the variable phase-center offset $\Delta_A \rho$ in cycles interpolated from the L1-band variable phase-center offset table for the receiver (the TOPEX satellite or a GPS receiving station on Earth) using Eqs. (11-61) to (11-63) be denoted by $\Delta_A \rho_{L1}$. Similarly, let the L2-band variable phase-center offset interpolated from the L2-band table be denoted by $\Delta_A \rho_{L2}$. Substituting the L1-band variable phase-center offset $\Delta_A \rho_{L1}/L1$ in seconds and the L2-band variable phase-center offset $\Delta_A \rho_{L2}/L2$ in seconds into Eqs. (7-2) to (7-4) gives the following

expression for the weighted average (WA) value of the variable phase-center offset $\Delta_A \rho(t_3)/f$ in Eq. (11-44):

$$\left[\frac{\Delta_A \rho(t_3)}{f} \right]_{\text{WA}} = \frac{L1 \Delta_A \rho(t_3)_{L1} - L2 \Delta_A \rho(t_3)_{L2}}{L1^2 - L2^2} \quad \text{s} \quad (11-64)$$

where $L1$ and $L2$ are given by Eq. (7-1). This value of $\Delta_A \rho(t_3)/f$ should be used in Eq. (11-44).

11.6 PRECISION QUASAR DELAY τ

Section 11.6.1 gives the definition of the precision quasar delay τ , and Section 11.6.2 gives the equation for calculating τ as a sum of terms. Most of the terms in this equation are calculated in the quasar light-time solution and in related calculations. Calculating τ as a sum of terms instead of the difference of two epochs reduces the roundoff errors in this calculation by approximately four orders of magnitude.

11.6.1 DEFINITION OF τ

The definition of the precision quasar delay τ is given by:

$$\tau = t_2(\text{ST})_{\text{R}} - t_1(\text{ST})_{\text{R}} \quad \text{s} \quad (11-65)$$

where $t_2(\text{ST})_{\text{R}}$ and $t_1(\text{ST})_{\text{R}}$ are reception times in station time ST of the quasar wavefront at the receiving electronics of receiver 2 and receiver 1, respectively. Each of these two receivers can be a tracking station on Earth or an Earth satellite. Substituting Eqs. (11-2) and (11-4) into Eq. (11-65) gives:

$$\tau = [t_2(\text{ST}) - t_1(\text{ST})] + \tau_{\text{D}_2} - \tau_{\text{D}_1} \quad \text{s} \quad (11-66)$$

where $t_2(\text{ST})$ and $t_1(\text{ST})$ are reception times in station time ST of the quasar wavefront at the tracking points of receivers 2 and 1, respectively. The various tracking points are defined in the fifth paragraph of Section 11.2. The quantities

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τ_{D_2} and τ_{D_1} are the downlink delays for receivers 2 and 1, respectively. If either receiver is an Earth satellite, its delay is currently set to zero.

11.6.2 CALCULATION OF τ

The precision quasar delay τ defined by Eq. (11–65) or (11–66) is calculated as the following sum of terms:

$$\begin{aligned}
 \tau = & \frac{r_{12}}{c} + RLT_{12} \\
 & - (ET - TAI)_{t_2} + (ET - TAI)_{t_1} \\
 & - (TAI - UTC)_{t_2} + (TAI - UTC)_{t_1} \\
 & - (UTC - ST)_{t_2} + (UTC - ST)_{t_1} \\
 & + \frac{1}{10^3 c} [\Delta_A \rho(t_2) + \Delta_{SC} \rho_2 - \Delta_A \rho(t_1) - \Delta_{SC} \rho_1] \\
 & + \tau_{D_2} - \tau_{D_1}
 \end{aligned} \tag{11-67}$$

where c is the speed of light in kilometers per second.

The distance r_{12} that the quasar wavefront travels from receiver 1 to receiver 2, the relativistic light-time delay RLT_{12} , the three time differences at the reception time t_2 at receiver 2, and the three time differences at the reception time t_1 at receiver 1 are all calculated in the quasar light-time solution as specified in Section 8.4.3.

In Eq. (11–67), the intermediate time UTC at receiver 2 or at receiver 1 is only used if that receiver is a DSN tracking station on Earth. If receiver 2 is an Earth satellite, UTC is replaced with TOPEX master time (TPX) and the constant offset (TAI – TPX) is obtained from the GIN file. Similarly, if receiver 1 is an Earth satellite, UTC is replaced with GPS master time (GPS) and the constant offset (TAI – GPS) is obtained from the GIN file.

The terms $\Delta_A \rho(t_2)$ and $\Delta_A \rho(t_1)$ are antenna corrections at receivers 2 and 1, respectively, if they are DSN tracking stations on Earth. They are calculated after the light-time solution from the formulation of Section 10.5. They are a function of the antenna type at the DSN tracking station, the axis offset b , and the secondary angle of the antenna. The value of this angle used to evaluate each antenna correction is one of the unrefracted auxiliary angles calculated at t_2 or t_1 from the formulation of Section 9. If either receiver is an Earth satellite, the analogous correction is the offset from the center of mass of the satellite to the nominal phase center of the satellite. This offset can be calculated as described in Section 7.3.3 when interpolating the ephemeris of the satellite, or it can be zero.

The down-leg solar corona range correction $\Delta_{SC} \rho_2$ at receiver 2 and the down-leg solar corona range correction $\Delta_{SC} \rho_1$ at receiver 1 are calculated in the quasar light-time solution from the formulation of Section 10.4.

The down-leg delay τ_{D_2} at receiver 2 and the down-leg delay τ_{D_1} at receiver 1 are obtained from the record of the OD file for the data point.

Equation (11–67) does not include corrections due to the troposphere and due to charged particles. These corrections are calculated in the Regres editor and are included in Eqs. (10–30) to (10–32) for the corrections $\Delta\tau$, $\Delta\tau_{\text{tr}}$ and $\Delta\tau_s$ to τ given by Eq. (11–67). These corrections to τ are handled separately as described in Sections 10.1 and 10.2. If either receiver is an Earth satellite, the troposphere and charged-particle corrections for that receiver are set to zero.